

Handout: More Differentiation Practice

Discussions 201, 203 // 2018-10-08

Problem 1 (Recycled from previous worksheet). Consider the function

$$f(x) = \begin{cases} x^2 \sin(1/x) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

- (1) Find the derivative $f'(x)$. (You calculated $f'(0)$ on the previous homework.)
 (2) Determine the limit

$$\lim_{x \rightarrow 0} f'(x).$$

Compare to the value of $f'(0)$.

Then answer the following true/false questions.

- _____ : f is defined at zero.
 _____ : f is continuous at zero.
 _____ : f is differentiable at zero.
 _____ : f' is defined at zero.
 _____ : f' is continuous at zero.
 _____ : f' is differentiable at zero.

Problem 2 (Stewart §3.2 #63). Find an expression for

$$\frac{d^n}{dx^n} e^x x^2$$

where n is a positive integer. (Compute the first few derivatives and then try to guess the general formula. Stewart suggests that you try to prove your guess is correct using mathematical induction.)

Problem 3. Let f and g be two functions related by

$$f(x) = e^x g(x).$$

Express $f^{(n)}(x)$ in terms of g and its derivatives. Check that your answer agrees with your answer to the preceding problem, in the case that $g(x) = x^2$.